

Mathematics-I Unit-3 Detailed Notes

RGPV Engineering

1. Convergence of Sequence and Series

Definition of Sequence:

A sequence is an ordered list of numbers.

Example:

1, 2, 3, 4, 5...

Convergent Sequence:

If terms approach a fixed value, then sequence is called convergent.

Example:

1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$... $\rightarrow$ approaches 0

Divergent Sequence:

If terms do not approach any fixed value, then sequence is divergent.

Example:

1, 2, 3, 4, 5...

Series:

Addition of sequence terms is called series.

Example:

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$...

This converges to 2.

2. Tests for Convergence

1. Comparison Test:

Compare given series with known series.

2. Ratio Test:

$L = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$

If $L < 1 \rightarrow$ Convergent

If $L > 1 \rightarrow$ Divergent

Example:

Series: $1 + 1/2 + 1/4 + 1/8 \dots$

Ratio = $1/2$

Since $1/2 < 1$,
Series is convergent.

3. Power Series

Definition:

Series containing powers of x is called power series.

General Form:

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

Example:

$$1 + x + x^2 + x^3 + \dots$$

This is a power series.

4. Taylor Series

Definition:

Taylor series expands function around $x=a$.

Formula:

$$f(x) = f(a) + (x-a)f'(a) + ((x-a)^2/2!)f''(a) + \dots$$

Example:

Expand e^x about $x=0$.

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots$$

5. Exponential Series

Formula:

$$e^x = 1 + x + x^2/2! + x^3/3! + \dots$$

Example:

Find first 4 terms of e^x .

Answer:

$$1 + x + x^2/2 + x^3/6$$

6. Trigonometric Series

Series for $\sin x$:

$$\sin x = x - x^3/3! + x^5/5! - \dots$$

Series for $\cos x$:

$$\cos x = 1 - x^2/2! + x^4/4! - \dots$$

Example:

Find first 3 terms of $\sin x$.

Answer:

$$x - x^3/6 + x^5/120$$

7. Logarithmic Series

Formula:

$$\log(1+x) = x - x^2/2 + x^3/3 - x^4/4 + \dots$$

Condition:

$$|x| < 1$$

Example:

Find first 4 terms.

Answer:

$$x - x^2/2 + x^3/3 - x^4/4$$

8. Fourier Series

Definition:

Any periodic function can be represented using sine and cosine terms.

General Formula:

$$f(x) = a_0/2 + \sum (a_n \cos nx + b_n \sin nx)$$

Applications:

Used in: Signal Processing Communication AI Electronics

9. Half Range Sine and Cosine Series

Half Range Sine Series:

Uses only sine terms.

Half Range Cosine Series:

Uses only cosine terms.

Applications:

Used in solving engineering problems.

10. Parseval's Theorem

Statement:

Parseval theorem relates Fourier coefficients with integration of square of function.

Formula:

$$\int f^2(x) dx = \pi [a_0^2/2 + \sum (a_n^2 + b_n^2)]$$

Applications:

Used in: Signal energy calculations Electrical Engineering Communication Systems